

## Contents

### Part I Characterization

<b>1 From Complex to Spatial Networks</b> .....	3
1.1 Early Days .....	3
1.2 Complex Networks .....	4
1.3 Space Matters .....	5
1.4 Definition and Representations .....	5
1.4.1 Spatial Networks .....	5
1.4.2 Representations of Networks .....	6
References .....	8
<b>2 Planar Graphs</b> .....	9
2.1 Graph Theoretical Tools .....	10
2.1.1 Definition and Representation .....	10
2.1.2 $K_5$ Is Not Planar .....	11
2.1.3 Euler's Formula .....	12
2.1.4 Distance from Planarity .....	13
2.2 Planarity of Street Networks .....	18
References .....	20
<b>3 Directed and Mixed Graphs</b> .....	23
3.1 Theoretical Results .....	23
3.1.1 Definitions .....	23
3.1.2 Components of a Digraph .....	24
3.1.3 Statistics of Loops .....	25
3.1.4 Perturbation of the Shortest Path in a Directed Lattice .....	27
3.2 Empirical Results: One-Way Streets .....	28
3.2.1 Fraction of One-Ways .....	30
3.2.2 Statistics of the Detour Index .....	31
3.2.3 Asymmetry of Shortest Paths .....	33
References .....	36

<b>4 Simple Measures .....</b>	<b>39</b>
4.1 Irrelevant Measures for Spatial Networks .....	39
4.1.1 Degree .....	40
4.1.2 Length of Segments .....	42
4.1.3 Cell Area .....	44
4.1.4 Clustering and Assortativity .....	45
4.1.5 Average Shortest Path and Diameter .....	47
4.1.6 Empirical Illustrations .....	48
4.2 Simple Characterizations .....	54
4.2.1 $\alpha$ and $\gamma$ Indices .....	54
4.2.2 Organic Ratio and Ringness .....	55
4.2.3 Edge Orientation Distribution .....	56
4.2.4 Shape Factor .....	58
4.2.5 Detour Index (or Stretch or Route Factor) .....	58
4.2.6 Cost, Efficiency and Robustness .....	60
References .....	63
<b>5 Betweenness Centrality .....</b>	<b>65</b>
5.1 Definition .....	66
5.2 General Properties .....	66
5.2.1 Numerical Calculation: Brandes' Algorithm .....	67
5.2.2 The Average BC .....	68
5.2.3 Edge Versus Node BC .....	69
5.2.4 Adding Edges .....	70
5.2.5 Relation Between the BC and the Clustering Coefficient .....	72
5.2.6 Scaling of the Maximum BC .....	72
5.3 The BC for Simple Graphs .....	74
5.3.1 Regular 1d and 2d Lattices .....	74
5.3.2 Cayley Tree .....	77
5.3.3 Branches and Ring .....	78
5.3.4 Grid-Tree Network .....	83
5.4 The BC in a Disk: The Continuous Limit .....	87
5.4.1 The Infinite Density Limit .....	87
5.4.2 Finite Density: A Perturbation Expansion .....	90
5.5 Empirical Measures on Street Networks .....	97
5.5.1 Spatial Patterns of Large BC Nodes .....	97
5.5.2 The BC and Socio-Economic Indicators .....	101
5.5.3 The BC Probability Distribution for Street Networks .....	101
5.5.4 The Effect of One-Way Streets on the BC .....	105
References .....	107

<b>6</b>	<b>The Shape of Shortest Paths</b>	109
6.1	(Euclidean) First Passage Percolation	109
6.1.1	Models and Definitions	109
6.1.2	Known Results About Exponents	112
6.1.3	Numerical Results	115
6.1.4	Travel Time and Transversal Fluctuations	117
	References	118
<b>7</b>	<b>Simplicity and Entropy</b>	121
7.1	Simplicity	121
7.1.1	Simplest Paths	121
7.1.2	The Simplicity Index and the Simplicity Profile	124
7.1.3	A Null Model	125
7.1.4	Measures on Real-World Networks	125
7.2	Information Perspective	130
7.2.1	Entropy and Simplest Paths	131
7.2.2	Quantifying the Complexity	133
	References	137
<b>8</b>	<b>Large-Scale Tools</b>	139
8.1	Spatial Dominance	140
8.1.1	Constructing the Dominance Tree	140
8.2	Class First-Passage Times	144
8.3	Community Detection in Spatial Networks	146
8.3.1	Modularity	147
8.3.2	A Null Model for Spatial Networks with Marks	148
8.3.3	Synthetic Spatial Network Benchmarks	152
8.3.4	Modifying the Modularity	153
8.4	Persistent Homology	157
8.4.1	Topological Data Analysis	157
8.4.2	Empirical Results	159
	References	165
<b>9</b>	<b>Typology of Planar Graphs</b>	167
9.1	Area and Shape of Faces	167
9.1.1	Characterizing Blocks	168
9.1.2	A Typology of Planar Graphs	171
9.2	Approximate Mapping of a Planar Graph to a Tree	174
9.3	An Exact Bijection Between a Planar Graph and a Tree	178
9.4	Machine Learning Approaches	182
	References	184
<b>10</b>	<b>Measuring the Time Evolution of Spatial Networks</b>	187
10.1	Road Networks	188
10.1.1	Organic Growth	188
10.1.2	Effect of Planning	196
10.1.3	Simplicity Measures	204

10.2	Subways .....	208
10.2.1	Generalities .....	208
10.2.2	Typical Numbers .....	210
10.2.3	Network Evolution .....	212
10.2.4	Standard Measures .....	213
10.2.5	Efficiency .....	214
10.2.6	Temporal Statistics: Bursts .....	217
10.2.7	Core and Branches: Measures and Model .....	219
10.2.8	Spatial Organization of the Core and Branches .....	227
	References .....	231

## Part II Models

<b>11</b>	<b>Spatial Generalizations of Random Graphs .....</b>	235
11.1	Spatial Version of Erdos-Renyi Graphs .....	235
11.1.1	The Erdos-Renyi Graph .....	235
11.1.2	Planar Erdos-Renyi Graphs .....	237
11.2	The Hidden Variable Model for Spatial Networks .....	238
11.2.1	Effect of Space .....	239
11.2.2	Effect of Traffic .....	240
	References .....	241
<b>12</b>	<b>Spatial Small-Worlds .....</b>	243
12.1	The Watts-Strogatz Model .....	243
12.2	Spatial Generalizations in Dimension $d$ .....	244
12.3	Navigability in the Kleinberg Model .....	247
12.3.1	Searchable Networks .....	247
12.3.2	Sketch of Kleinberg's Proof .....	249
	References .....	252
<b>13</b>	<b>Growing Spatial Networks .....</b>	253
13.1	Preferential Attachment and Space .....	253
13.1.1	Preferential Attachment and Distance Selection .....	255
13.1.2	Searching in Spatial Scale-Free Networks .....	262
13.2	Attraction Potential Models .....	263
13.2.1	The Connection Rule .....	264
13.2.2	Uniform Distribution of Nodes .....	265
13.2.3	Exponential Distribution of Centers .....	266
13.2.4	Effect of Centrality and Density .....	267
13.2.5	The Appearance of Core Districts .....	275
	References .....	276
<b>14</b>	<b>Tessellations of the Plane .....</b>	277
14.1	The Voronoi Tessellation .....	277
14.1.1	Average Properties of the Poisson-Voronoi Tessellation .....	279
14.1.2	Statistical Properties .....	281

14.1.3	Central Limit Theorem for the Total Length .....	285
14.2	Effect of the Density of Points .....	286
14.3	Crack and STIT Tessellations .....	288
14.4	Planar Fragmentation .....	289
	References .....	292
<b>15</b>	<b>Proximity Graphs .....</b>	<b>295</b>
15.1	Random Geometric Graphs .....	296
15.1.1	The Hard Case .....	296
15.1.2	Soft Random Geometric Graphs .....	301
15.1.3	The Full Connectivity Probability .....	302
15.1.4	The Waxman Model .....	304
15.1.5	Random Geometric Graphs in Hyperbolic Space .....	308
15.2	Bluetooth Graph and Sparsification .....	309
15.3	The $k$ -nearest Neighbour Model .....	310
15.3.1	Definition and Connectivity Properties .....	310
15.3.2	A Scale-Free Network on a Lattice .....	311
15.4	A Dynamical Proximity Model .....	313
15.4.1	The Model .....	313
15.4.2	Stationary State .....	314
15.4.3	Percolation Properties .....	315
15.4.4	Degree Distribution .....	315
15.5	Apollonian Networks .....	316
	References .....	317
<b>16</b>	<b>Excluded Volume Graphs .....</b>	<b>319</b>
16.1	Delaunay Graph .....	319
16.2	Gabriel Graph .....	320
16.3	Relative Neighborhood Graph .....	323
16.4	$\beta$ -Skeleltons .....	323
	References .....	326
<b>17</b>	<b>Loops and Branches .....</b>	<b>327</b>
17.1	Reducing the Complexity of a Spatial Network .....	327
17.2	A Loop and Branches Toy Model .....	330
17.2.1	Exact and Approximate Formulas for the BC .....	331
17.2.2	Threshold Value of $w$ and Optimal $\ell$ .....	335
17.3	Analyzing the Impact of Congestion Cost .....	339
17.3.1	An Exactly Solvable Hub-and-Spoke Model .....	339
17.3.2	Congestion and Centralized Organization .....	343
	References .....	346
<b>18</b>	<b>Optimal Networks .....</b>	<b>347</b>
18.1	Optimization, Complexity, and Efficiency .....	347
18.1.1	Complexity .....	347
18.1.2	Efficiency of Transport Network .....	348
18.2	Minimum Spanning Tree .....	350

18.2.1	Minimum Spanning Tree on a Complete Graph .....	351
18.2.2	Properties of the Euclidean Minimum Spanning Tree .....	353
18.3	Geometric $t$ -Spanners .....	359
18.3.1	Definition .....	359
18.3.2	The Theta Graph .....	360
18.4	Optimal Trees: Generalization .....	361
18.5	Beyond Optimal Trees: Noise and Loops .....	366
	References .....	369
<b>19</b>	<b>Optimal Transportation Networks and Network Design</b> .....	373
19.1	Empirical Motivation: The Structure of Subway Networks .....	374
19.2	Hub-and-Spoke Structure .....	376
19.3	One-Dimensional Problems .....	379
19.3.1	A Single Open Line .....	379
19.3.2	Transition for a Cyclic Service Line .....	383
19.4	Multiple Transit Lines .....	386
19.4.1	Parallel Transit Lines .....	386
19.4.2	Radial Lines .....	388
19.5	The Optimal Subway Problem .....	392
19.5.1	A First Simplification: Optimal Placement .....	393
19.5.2	The Minimum Average Distance to the Center .....	397
19.5.3	Average Minimum Time Between All Pairs of Points .....	400
	References .....	405
<b>20</b>	<b>Greedy Models</b> .....	407
20.1	A Model for Distribution Networks .....	408
20.2	Cost-Benefit Analysis .....	410
20.2.1	Theoretical Formulation .....	411
20.2.2	Crossover Between the Star Graph and the MST .....	412
20.2.3	Spatial Hierarchy and Scaling .....	415
20.2.4	Understanding the Scaling with a Toy Model .....	418
20.2.5	Efficiency .....	419
20.3	Cost-Benefit Analysis: General Scaling Theory .....	422
20.3.1	Theoretical Framework .....	424
20.3.2	Subways .....	425
20.3.3	Railways .....	429
	References .....	433
<b>Index</b>	.....	435