Table of Contents

1. Schauder Bases 1

   a. Existence of Bases and Examples 1
   b. Schauder Bases and Duality 7
   c. Unconditional Bases 15
   d. Examples of Spaces Without an Unconditional Basis 24
   e. The Approximation Property 29
   f. Biorthogonal Systems 42
   g. Schauder Decompositions 47

2. The Spaces c_0 and l_p 53

   a. Projections in c_0 and l_p and Characterizations of these Spaces . . 53
   b. Absolutely Summing Operators and Uniqueness of Unconditional Bases 63
   c. Fredholm Operators, Strictly Singular Operators and Complemented Subspaces of $l_p \oplus l_r$ 75
   d. Subspaces of c_0 and l_p and the Approximation Property, Complementably Universal Spaces 84
   e. Banach Spaces Containing l_p or c_0 95
   f. Extension and Lifting Properties, Automorphisms of l_∞, c_0 and l_1 . 104

3. Symmetric Bases 113

   a. Properties of Symmetric Bases, Examples and Special Block Bases . 113
   b. Subspaces of Spaces with a Symmetric Basis 123

4. Orlicz Sequence Spaces 137

   a. Subspaces of Orlicz Sequence Spaces which have a Symmetric Basis . 137
   b. Duality and Complemented Subspaces 147
   c. Examples of Orlicz Sequence Spaces 156
   d. Modular Sequence Spaces and Subspaces of $l_p \oplus l_r$ 166
   e. Lorentz Sequence Spaces 175

References 180

Subject Index 185

Table of Contents

I. Banach Lattices 1

a. Basic Definitions and Results 1

Characterizations of σ-completeness and σ-order continuity. Ideals, bands and band projections. Boolean algebras of projections and cyclic spaces.

b. Concrete Representation of Banach Lattices 14

Abstract L_p and M spaces. Joint characterizations of $L_p(\mu)$ and $c_0(\Gamma)$ spaces. The functional representation theorem for order continuous Banach lattices. Köthe function spaces. The Fatou property.

c. The Structure of Banach Lattices and their Subspaces 31

Property (u). Weak completeness and reflexivity. Existence of unconditional basic sequences.

d. p-Convexity in Banach Lattices 40

Functional calculus in general Banach lattices. The definition and basic properties of p-convexity and p-concavity. Duality. The p-convexification and p-concavification procedures. Some connections with p-absolutely summing operators. Factorization through L_p spaces.

e. Uniform Convexity in General Banach Spaces and Related Notions. 59

The definition of the moduli of convexity and smoothness. Duality. Asymptotic behavior of the moduli. The moduli of $L_2(X)$. Convergence of series in uniformly convex and uniformly smooth spaces. The notions of type and cotype. Kahane's theorem. Connections between the moduli and type and cotype.

f. Uniform Convexity in Banach Lattices and Related Notions . . 79

Uniformly convex and smooth renormings of a $(p>1)$-convex and $(q<\infty)$-concave Banach lattice. The concepts of upper and lower estimates for disjoint elements. The relations between these notions and those of type, cotype, p-convexity, q-concavity, p-absolutely summing operators, etc. Examples. Two diagrams summarizing the various connections.

g. The Approximation Property and Banach Lattices. 102

Examples of Banach lattices and of subspaces of l_p, $p\neq 2$, without the B.A.P. The connection between the type and the cotype of a space and the existence of subspaces without the B.A.P. A space different from l_2 all of whose subspaces have the B.A.P.

II. Rearrangement Invariant Function Spaces 114

a. Basic Definitions, Examples and Results 114

The definition of r.i. function spaces on $[0,1]$ and $[0,\infty)$. Conditional expectations. Interpolation between L_1 and L_∞.

b. The Boyd Indices. 129

The definition of Boyd indices. Duality. The Rademacher functions in an r.i. function space on [0, 1]. The characterization of Boyd indices in terms of existence of l_p^n's for all n on disjoint vectors having the same distribution. Operators of weak type (p, q). Boyd's interpolation theorem.

c. The Haar and the Trigonometric Systems 150

Basic results on martingales. The unconditionality of the Haar system in $L_p(0, 1)$ spaces $(1<p<\infty)$ and in more general r.i. function spaces on [0, 1]. Reproducibility of bases. The boundedness of the Riesz projection and applications.

d. Some Results on Complemented Subspaces 168

The isomorphism between an r.i. function space X on [0, 1] with non-trivial Boyd indices and the spaces $X(l_2)$ and Rad X. Complemented subspaces of X with an unconditional basis. Subspaces spanned by a subsequence of the Haar system. R.i. spaces with non-trivial Boyd indices are primary.

e. Isomorphisms Between r.i. Function Spaces; Uniqueness of the r.i. Structure 181

Isomorphic embeddings. Classification of symmetric basic sequences in r.i. function spaces of type 2. Uniqueness of the r.i. structure for $L_p(0, 1)$ spaces and for $(q<2)$-concave r.i. spaces. Other applications.

f. Applications of the Poisson Process to r.i. Function Spaces . . . 207

The isomorphism between $L_p(0, 1)$ and $L_p(0, \infty)\cap L_2(0, \infty)$ for $p>2$. R.i. function spaces in $[0, \infty)$ isomorphic to a given r.i. function space on [0, 1]. Isometric embeddings of $L_r(0,1)$ into $L_p(0,1)$ for $1\leqslant p<r<2$ and in other r.i. function spaces. The complementation of the spaces $X_{p,2}$ in $L_p(0,1)$, $p>2$ and generalizations.

g. Interpolation Spaces and their Applications 213

Interpolation pairs. General interpolation spaces and applications to the construction of r.i. function spaces without unique r.i. structure. The Lions–Peetre method of interpolation. Uniform convexity and type in interpolation spaces.

References 233

Subject Index 239