

Contents

| | |
|---|----|
| 1 Markov Chains | 1 |
| 1.1 Markov Kernels | 1 |
| 1.2 Markov Chains | 2 |
| 1.3 The Canonical Chain | 5 |
| 1.4 Markov and Strong Markov Properties | 6 |
| 1.5 Continuous Time: Markov Processes | 8 |
| 2 Countable Markov Chains | 11 |
| 2.1 Recurrence and Transience | 11 |
| 2.1.1 Positive Recurrence | 15 |
| 2.1.2 Null Recurrence | 19 |
| 2.2 Subsets of Recurrent Sets | 20 |
| 2.3 Recurrence and Lyapunov Functions | 24 |
| 2.4 Aperiodic Chains | 26 |
| 2.5 The Convergence Theorem | 28 |
| 2.6 Application to Renewal Theory | 31 |
| 2.6.1 Coupling of Renewal Processes | 33 |
| 2.7 Convergence Rates for Positive Recurrent Chains | 35 |
| 3 Random Dynamical Systems | 37 |
| 3.1 General Definitions | 37 |
| 3.2 Representation of Markov Chains by RDS | 39 |
| 4 Invariant and Ergodic Probability Measures | 47 |
| 4.1 Weak Convergence of Probability Measures | 47 |
| 4.1.1 Tightness and Prohorov's Theorem | 53 |
| 4.2 Invariant Measures | 55 |
| 4.2.1 Tightness Criteria for Empirical Occupation Measures | 57 |
| 4.3 Excessive Measures | 59 |
| 4.4 Ergodic Measures | 59 |

| | | |
|----------|---|------------|
| 4.5 | Unique Ergodicity | 62 |
| 4.5.1 | Unique Ergodicity of Random Contractions | 63 |
| 4.6 | Classical Results from Ergodic Theory | 67 |
| 4.6.1 | Poincaré, Birkhoff, and Ergodic Decomposition Theorems | 69 |
| 4.7 | Application to Markov Chains | 73 |
| 4.8 | Continuous Time: Invariant Probabilities for Markov Processes | 79 |
| 5 | Irreducibility | 85 |
| 5.1 | Resolvent and ξ -Irreducibility | 85 |
| 5.2 | The Accessible Set | 87 |
| 5.2.1 | Continuous Time: Accessibility | 91 |
| 5.3 | The Asymptotic Strong Feller Property | 92 |
| 5.3.1 | Strong Feller Implies Asymptotic Strong Feller | 93 |
| 5.3.2 | A Sufficient Condition for the Asymptotic Strong Feller Property | 98 |
| 5.3.3 | Unique Ergodicity of Asymptotic Strong Feller Chains | 101 |
| 6 | Petite Sets and Doeblin Points | 111 |
| 6.1 | Petite Sets, Small Sets, Doeblin Points | 111 |
| 6.1.1 | Continuous Time: Doeblin Points for Markov Processes | 112 |
| 6.2 | Random Dynamical Systems | 114 |
| 6.3 | Random Switching Between Vector Fields | 119 |
| 6.3.1 | The Weak Bracket Condition | 121 |
| 6.4 | Piecewise Deterministic Markov Processes | 125 |
| 6.4.1 | Invariant Measures | 127 |
| 6.4.2 | The Strong Bracket Condition | 130 |
| 6.5 | Stochastic Differential Equations | 135 |
| 6.5.1 | Accessibility | 136 |
| 6.5.2 | Hörmander Conditions | 136 |
| 7 | Harris and Positive Recurrence | 143 |
| 7.1 | Stability and Positive Recurrence | 143 |
| 7.2 | Harris Recurrence | 145 |
| 7.2.1 | Petite Sets and Harris Recurrence | 146 |
| 7.3 | Recurrence Criteria and Lyapunov Functions | 147 |
| 7.4 | Subsets of Recurrent Sets | 150 |
| 7.5 | Petite Sets and Positive Recurrence | 152 |
| 7.6 | Positive Recurrence for Feller Chains | 156 |
| 7.6.1 | Application to PDMPs | 157 |
| 7.6.2 | Application to SDEs | 159 |

| | |
|---|-----|
| 8 Harris Ergodic Theorem | 161 |
| 8.1 Total Variation Distance | 161 |
| 8.1.1 Coupling | 163 |
| 8.2 Harris Convergence Theorems | 164 |
| 8.2.1 Geometric Convergence | 164 |
| 8.2.2 Continuous Time: Exponential Convergence | 169 |
| 8.2.3 Coupling, Splitting, and Polynomial Convergence | 170 |
| 8.3 Convergence in Wasserstein Distance | 175 |
| A Monotone Class and Martingales | 181 |
| A.1 Monotone Class Theorem | 181 |
| A.2 Conditional Expectation | 181 |
| A.3 Martingales | 183 |
| Bibliography | 187 |
| List of Symbols | 191 |
| Index | 195 |