Contents

Preface v

Part I Compact Groups

1 Haar Measure 3

2 Schur Orthogonality 7

3 Compact Operators 19

4 The Peter–Weyl Theorem 23

Part II Compact Lie Groups

5 Lie Subgroups of $\mathrm{GL}(n, \mathbb{C})$ 31

6 Vector Fields 39

7 Left-Invariant Vector Fields 45

8 The Exponential Map 51

9 Tensors and Universal Properties 57

10 The Universal Enveloping Algebra 61

11 Extension of Scalars 67

12 Representations of $\mathfrak{sl}(2, \mathbb{C})$ 71

13 The Universal Cover 81

14 The Local Frobenius Theorem 93

15 Tori 101

16 Geodesics and Maximal Tori 109

17 The Weyl Integration Formula 123

18 The Root System 129

19 Examples of Root Systems 145

20 Abstract Weyl Groups 157

21 Highest Weight Vectors 169

22 The Weyl Character Formula 177

23 The Fundamental Group 191

Part III Noncompact Lie Groups

24 Complexification 205

25 Coxeter Groups 213

26 The Borel Subgroup 227

27 The Bruhat Decomposition 243

28 Symmetric Spaces 257

29 Relative Root Systems 281

30 Embeddings of Lie Groups 303

31 Spin 319

Part IV Duality and Other Topics

32 Mackey Theory 337

33 Characters of $\mathrm{GL}(n, \mathbb{C})$ 349

34 Duality Between S_k and $\mathrm{GL}(n, \mathbb{C})$ 355

35 The Jacobi–Trudi Identity 365

36 Schur Polynomials and $\mathrm{GL}(n, \mathbb{C})$ 379

37 Schur Polynomials and S_k 387

38 The Cauchy Identity 395

39 Random Matrix Theory 407

40 Symmetric Group Branching Rules and Tableaux 419

41 Unitary Branching Rules and Tableaux 427

42 Minors of Toeplitz Matrices 437

43 The Involution Model for S_k 445

44 Some Symmetric Algebras 455

45 Gelfand Pairs 461

46 Hecke Algebras 471

47 The Philosophy of Cusp Forms 485

48 Cohomology of Grassmannians 517

Appendix: Sage 529

References 535

Index 545