

Contents

CONTENTS

1 Basic Properties of Control Systems	1
1.1 Basic Definitions	1
1.2 Systems Given by Differential Equations	8
1.3 Some Control-Theoretic Notions	13
1.4 Control Flow and Control Sets	16
1.5 Linearization and Regular Trajectories	34
1.6 Comments and Bibliographical Notes	42
2 Introduction to Invariance Entropy	43
2.1 Definitions and Basic Properties	43
2.2 Elementary Properties	49
2.3 A One-Dimensional Linear Example	66
2.4 Relations to Topological Feedback Entropy	68
2.5 Relations to Minimal Data Rates	78
2.6 Comments and Bibliographical Notes	86
3 Linear and Bilinear Systems	89
3.1 Linear Systems	90
3.2 Inhomogeneous Bilinear Systems	96
3.3 Comments and Bibliographical Notes	105
4 General Estimates	107
4.1 Upper Bounds of Ito-Type	107
4.2 Lower Bounds in Terms of Volume Growth Rates	117
4.3 Comments and Bibliographical Notes	120
5 Controllability, Lyapunov Exponents, and Upper Bounds	121
5.1 The Upper Bound Theorem for Control Sets	122
5.2 Approximation Results for Lyapunov Exponents	137
5.3 Comments and Bibliographical Notes	150
6 Escape Rates and Lower Bounds	151
6.1 Escape Rates and Invariance Entropy	151
6.2 The First Lower Bound Theorem	153

6.3	The Second Lower Bound Theorem	167
6.4	Comments and Bibliographical Notes	175
7	Examples	177
7.1	One-Dimensional Control-Affine Systems	177
7.2	Uniformly Expanding Systems	184
7.3	Inhomogeneous Bilinear Systems Revisited	192
7.4	Projective Systems	202
7.5	Comments and Bibliographical Notes	219
A	General Concepts	221
A.1	Linear and Multilinear Algebra	221
A.2	Differentiable Manifolds	225
A.3	Carathéodory Differential Equations	236
A.4	Metric Spaces	242
B	Dynamical Systems	245
B.1	Chain Recurrence and Chain Transitivity	245
B.2	Vector Bundles and Linear Flows	247
B.3	Dimension Theory and Topological Entropy	252
B.4	Additive and Subadditive Cocycles	261
	References	263
	Index	269