
Contents

	<i>Preface</i>	<i>page</i> vii
1	Probability spaces	1
	1.1 Discrete examples	1
	1.2 Probability spaces	6
	1.3 Lebesgue measure	11
	1.4 Lebesgue integral	13
	1.5 Lebesgue outer measure	33
2	Probability distributions and random variables	39
	2.1 Probability distributions	39
	2.2 Random variables	46
	2.3 Expectation and variance	56
	2.4 Moments and characteristic functions	62
3	Product measure and independence	66
	3.1 Product measure	67
	3.2 Joint distribution	73
	3.3 Iterated integrals	75
	3.4 Random vectors in \mathbb{R}^n	81
	3.5 Independence	83
	3.6 Covariance	96
	3.7 Proofs by means of d -systems	98
4	Conditional expectation	106
	4.1 Binomial stock prices	106
	4.2 Conditional expectation: discrete case	112
	4.3 Conditional expectation: general case	119
	4.4 The inner product space $L^2(P)$	130
	4.5 Existence of $\mathbb{E}(X \mathcal{G})$ for integrable X	137
	4.6 Proofs	142
5	Sequences of random variables	147
	5.1 Sequences in $L^2(P)$	147
	5.2 Modes of convergence for random variables	156
	5.3 Sequences of i.i.d. random variables	167
	5.4 Convergence in distribution	170
	5.5 Characteristic functions and inversion formula	174

5.6	Limit theorems for weak convergence	176
5.7	Central Limit Theorem	180
	<i>Index</i>	187