
Contents

Preface	ix
1 Results from One-Variable Calculus	1
1.1 The Real Number System	1
1.2 Foundational and Basic Theorems	4
1.3 Taylor's Theorem	6

Part I Multivariable Differential Calculus

2 Euclidean Space	23
2.1 Algebra: Vectors	23
2.2 Geometry: Length and Angle	31
2.3 Analysis: Continuous Mappings	41
2.4 Topology: Compact Sets and Continuity	51
3 Linear Mappings and Their Matrices	59
3.1 Linear Mappings	60
3.2 Operations on Matrices	72
3.3 The Inverse of a Linear Mapping	79
3.4 Inhomogeneous Linear Equations	87
3.5 The Determinant: Characterizing Properties and Their Consequences	88
3.6 The Determinant: Uniqueness and Existence	97
3.7 An Explicit Formula for the Inverse	108
3.8 Geometry of the Determinant: Volume	110
3.9 Geometry of the Determinant: Orientation	120
3.10 The Cross Product, Lines, and Planes in \mathbb{R}^3	122

4	The Derivative	131
4.1	Trying to Extend the Symbol-Pattern: Immediate, Irreparable Catastrophe	132
4.2	New Environment: The Bachmann–Landau Notation	132
4.3	One-Variable Revisionism: The Derivative Redefined	140
4.4	Basic Results and the Chain Rule	146
4.5	Calculating the Derivative	153
4.6	Higher-Order Derivatives	165
4.7	Extreme Values	175
4.8	Directional Derivatives and the Gradient	185
5	Inverse and Implicit Functions	199
5.1	Preliminaries	201
5.2	The Inverse Function Theorem	206
5.3	The Implicit Function Theorem	213
5.4	Lagrange Multipliers: Geometric Motivation and Specific Examples	229
5.5	Lagrange Multipliers: Analytic Proof and General Examples ..	240

Part II Multivariable Integral Calculus

6	Integration	253
6.1	Machinery: Boxes, Partitions, and Sums	253
6.2	Definition of the Integral	263
6.3	Continuity and Integrability	269
6.4	Integration of Functions of One Variable	277
6.5	Integration over Nonboxes	285
6.6	Fubini’s Theorem	294
6.7	Change of Variable	307
6.8	Topological Preliminaries for the Change of Variable Theorem	328
6.9	Proof of the Change of Variable Theorem	335
7	Approximation by Smooth Functions	349
7.1	Spaces of Functions	351
7.2	Pulse Functions	356
7.3	Convolution	358
7.4	Test Approximate Identity and Convolution	364
7.5	Known-Integrable Functions	370
8	Parametrized Curves	375
8.1	Euclidean Constructions and Two Curves	375
8.2	Parametrized Curves	384
8.3	Parametrization by Arc Length	390
8.4	Plane Curves: Curvature	394

8.5	Space Curves: Curvature and Torsion	398
8.6	General Frenet Frames and Curvatures	404
9	Integration of Differential Forms	409
9.1	Integration of Functions over Surfaces	410
9.2	Flow and Flux Integrals	418
9.3	Differential Forms Syntactically and Operationally	424
9.4	Examples: 1-Forms	427
9.5	Examples: 2-Forms on \mathbb{R}^3	430
9.6	Algebra of Forms: Basic Properties	437
9.7	Algebra of Forms: Multiplication	439
9.8	Algebra of Forms: Differentiation	441
9.9	Algebra of Forms: The Pullback	447
9.10	Change of Variable for Differential Forms	459
9.11	Closed Forms, Exact Forms, and Homotopy	461
9.12	Cubes and Chains	467
9.13	Geometry of Chains: The Boundary Operator	469
9.14	The General Fundamental Theorem of Integral Calculus	476
9.15	Classical Change of Variable Revisited	481
9.16	The Classical Theorems	487
9.17	Divergence and Curl in Polar Coordinates	493
	Index	503