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This book contains 1000 problems on infinite series and sequences, with detailed solutions. It is intended for students of mathematics, physics, and engineering, as well as for teachers and engineers who are interested in the theory of infinite series and sequences.

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