
Contents

1	Sequences and Series of Functions	1
1.1	Sequences of Functions: Pointwise and Uniform Convergence....	1
1.2	First Theorems on Uniform Convergence	4
1.3	Theorems on Interchanging Limits and Integrals or Derivatives...	7
1.4	Uniform Convergence and Monotonicity	14
1.5	Series of Functions	17
1.6	Power Series	22
1.7	Taylor Series	28
1.8	Fourier Series	36
1.9	The Convergence of Fourier Series	42
	Appendix to Chap. 1	48
1.10	The Ascoli-Arzelà Theorem	48
1.11	The Weierstrass Approximation Theorem	50
1.12	Abel's Theorem on Power Series	52
2	Metric Spaces and Banach Spaces	59
2.1	Introduction	59
2.2	Metric Spaces	59
2.3	Sequences in a Metric Space: Continuous Functions	65
2.4	Vector Spaces: Linear Maps	69
2.5	The Vector Space \mathbb{R}^n and Its Dual	72
2.6	Normed Vector Spaces	76
2.7	The Normed Vector Space \mathbb{R}^n	78
2.8	Complete Metric Spaces: Banach Spaces	83
2.9	Lipschitz Functions: The Contraction Theorem	86
2.10	Compact Sets: Continuous Functions on Compact Sets	89
2.11	Connected Open Subsets of \mathbb{R}^n	92
	Appendix to Chap. 2	94
2.12	Further Compactness Theorems: Generalised Weierstrass Theorem	94

3	Functions of Several Variables	101
3.1	Round-Up of Topology in \mathbb{R}^n	101
3.2	Limits and Continuity	103
3.3	Partial Derivatives	105
3.4	Higher Derivatives. Schwarz's Theorem	109
3.5	Gradient. Differentiability	113
3.6	Composite Functions	118
3.7	Directional Derivatives	122
3.8	Functions with Vanishing Gradient on Connected Sets	127
3.9	Homogeneous Functions	129
3.10	Functions Defined by Integrals	131
3.11	Taylor Formula and Higher-Order Differentials	135
3.12	Quadratic Forms. Definite, Semi-definite and Indefinite Matrices	140
3.13	Local Maxima and Minima	144
3.14	Vector-Valued Functions	150
	Appendix to Chap. 3	158
3.15	Convex Functions	158
3.16	Complements on Quadratic Forms	173
3.17	The Maximum Principle for Harmonic Functions	181
4	Ordinary Differential Equations	187
4.1	Introduction: The Initial Value Problem	187
4.2	Cauchy's Local Existence and Uniqueness Theorem	196
4.3	First Consequences of Cauchy's Theorem	206
4.4	The Global Existence and Uniqueness Theorem: Extension of Solutions	210
4.5	Solving First-Order ODEs in Normal Form	216
4.6	Solving First-Order ODEs Not in Normal Form	221
4.7	Solving Higher-Order Equations	224
4.8	Qualitative Study of Solutions	226
	Appendix to Chap. 4	232
4.9	Peano's Theorem	232
5	Linear Differential Equations	237
5.1	General Properties	237
5.2	General Integral of Linear ODEs	241
5.3	The Method of Variation of Parameters	247
5.4	Bernoulli Equations	250
5.5	Homogeneous Equations with Constant Coefficients	252
5.6	Equations with Constant Coefficients and Special Right-Hand Side	257
5.7	Linear Euler Equations	260
	Appendix to Chap. 5	263
5.8	Boundary Value Problems	263
5.9	Linear Systems	268

6	Curves and Integrals Along Curves	273
6.1	Regular Curves	273
6.2	Oriented Curves	279
6.3	The Length of a Curve	281
6.4	The Integral of a Function Along a Curve	286
6.5	The Curvature of a Plane Curve	290
6.6	The Cross Product in \mathbb{R}^3	294
6.7	Biregular Curves in \mathbb{R}^3 : Curvature	297
	Appendix to Chap. 6	300
6.8	Curves in \mathbb{R}^3 : Torsion, Frenet Frame	300
7	Differential One-Forms	305
7.1	Vector Fields. Work. Conservative Fields	305
7.2	Differential 1-Forms. Line Integrals	308
7.3	Exact 1-Forms	311
7.4	Exact 1-Forms on the Plane. Simply Connected Open Sets in \mathbb{R}^2	315
7.5	One-Forms in Space. Irrotational Vector Fields	320
	Appendix to Chap. 7	323
7.6	Simply Connected Open Sets in \mathbb{R}^n and Exact 1-Forms	323
8	Multiple Integrals	325
8.1	Double Integrals on Normal Domains	325
8.2	Reduction Formulas for Double Integrals	335
8.3	Gauss-Green Formulas. The Divergence Theorem. Stokes's Formula	342
8.4	Variable Change in Double Integrals	351
8.5	Triple Integrals	356
8.6	Peano-Jordan Measurable Subsets of \mathbb{R}^n	362
8.7	The Riemann Integral in \mathbb{R}^n	369
8.8	Properties of Riemann Integrals	377
8.9	Summable Functions	382
	Appendix to Chap. 8	388
8.10	Jensen's Inequality	388
8.11	The Gamma Function. The Measure of the Unit Ball in \mathbb{R}^n	389
9	The Lebesgue Integral	395
9.1	Introduction	395
9.2	Pluri-Intervals. Open Sets. Compact Sets	396
9.3	Bounded Measurable Sets	401
9.4	Unbounded Measurable Sets	405
9.5	Measurable Functions	412
9.6	The Lebesgue Integral. Interchanging Limits and Integrals	419
9.7	Measure and Integration on Product Spaces	437
9.8	Changing Variables in Multiple Integrals	457
	Appendix to Chap. 9	477

9.9	L^p Spaces	477
9.10	Differentiability of Monotone Functions	485
9.11	Functions with Bounded Variation	495
9.12	Absolutely Continuous Functions	504
9.13	The Indefinite Integral in Lebesgue's Theory	514
10	Surfaces and Surface Integrals	525
10.1	Regular Surfaces	525
10.2	Local Coordinates and Change of Parameters	533
10.3	The Tangent Plane and the Unit Normal	539
10.4	The Area of a Surface	543
10.5	Orientable Surfaces: Surfaces with Boundary	550
10.6	Surface Integrals	556
10.7	Stokes's Formula and the Divergence Theorem	560
11	Implicit Functions	567
11.1	The Implicit Function Theorem for Equations	567
11.2	The Implicit Function Theorem for Systems	582
11.3	Local and Global Invertibility	589
11.4	Constrained Maxima and Minima. Lagrange Multipliers	596
	Appendix to Chap. 11	606
11.5	Singular Points of a Plane Curve	606
12	Manifolds in \mathbb{R}^n and k-Forms	611
12.1	k -Dimensional Manifolds in \mathbb{R}^n	611
12.2	The Tangent Space and the Normal Space of a Manifold	619
12.3	Measure and Integration on k -Submanifolds in \mathbb{R}^n	624
12.4	The Divergence Theorem	632
12.5	Alternating Forms	638
12.6	Differential k -Forms	645
12.7	Orientable Manifolds. Integration of k -Forms on Manifolds	650
12.8	Manifolds with Boundary. Stokes's Formula	659
	Appendix to Chap. 12	663
12.9	Exact and Closed Differential Forms	663
	Index	669