

Contents

I	Fourier Analysis	3
1	Fourier Series	5
1.1	Periodic Functions	5
1.2	Exponentials	7
1.3	The Bessel Inequality	9
1.4	Convergence in the L^2 -Norm	10
1.5	Uniform Convergence of Fourier Series	17
1.6	Periodic Functions Revisited	19
1.7	Exercises	19
2	Hilbert Spaces	25
2.1	Pre-Hilbert and Hilbert Spaces	25
2.2	ℓ^2 -Spaces	29
2.3	Orthonormal Bases and Completion	31
2.4	Fourier Series Revisited	36
2.5	Exercises	37
3	The Fourier Transform	41
3.1	Convergence Theorems	41
3.2	Convolution	43
3.3	The Transform	46

3.4	The Inversion Formula	49
3.5	Plancherel's Theorem	52
3.6	The Poisson Summation Formula	54
3.7	Theta Series	56
3.8	Exercises	56
4	Distributions	59
4.1	Definition	59
4.2	The Derivative of a Distribution	61
4.3	Tempered Distributions	62
4.4	Fourier Transform	65
4.5	Exercises	68
II	LCA Groups	71
5	Finite Abelian Groups	73
5.1	The Dual Group	73
5.2	The Fourier Transform	75
5.3	Convolution	77
5.4	Exercises	78
6	LCA Groups	81
6.1	Metric Spaces and Topology	81
6.2	Completion	89
6.3	LCA Groups	94
6.4	Exercises	96
7	The Dual Group	101
7.1	The Dual as LCA Group	101

7.2	Pontryagin Duality	107
7.3	Exercises	108
8	Plancherel Theorem	111
8.1	Haar Integration	111
8.2	Fubini's Theorem	116
8.3	Convolution	120
8.4	Plancherel's Theorem	122
8.5	Exercises	125
III	Noncommutative Groups	127
9	Matrix Groups	129
9.1	$GL_n(\mathbb{C})$ and $U(n)$	129
9.2	Representations	131
9.3	The Exponential	133
9.4	Exercises	138
10	The Representations of $SU(2)$	141
10.1	The Lie Algebra	142
10.2	The Representations	146
10.3	Exercises	147
11	The Peter-Weyl Theorem	149
11.1	Decomposition of Representations	149
11.2	The Representation on $\text{Hom}(V_\gamma, V_\tau)$	150
11.3	The Peter-Weyl Theorem	151
11.4	A Reformulation	154
11.5	Exercises	155

12 The Heisenberg Group	157
12.1 Definition	157
12.2 The Unitary Dual	158
12.3 Hilbert-Schmidt Operators	162
12.4 The Plancherel Theorem for \mathcal{H}	167
12.5 A Reformulation	169
12.6 Exercises	173
A The Riemann Zeta Function	175
B Haar Integration	179
Bibliography	187
Index	190