
Contents

1	Introduction	1
1.1	Summary	1
1.2	Finance.....	1
1.3	Econometrics	3
1.4	Mathematics	6
1.5	Problems and Solutions	7
	References.....	10
 Part I Time Series Modeling		
2	Basic Concepts from Probability Theory	13
2.1	Summary	13
2.2	Random Variables	13
2.3	Joint and Conditional Distributions	22
2.4	Stochastic Processes (SP)	29
2.5	Problems and Solutions	35
	References.....	42
3	Autoregressive Moving Average Processes (ARMA)	45
3.1	Summary	45
3.2	Moving Average Processes	45
3.3	Lag Polynomials and Invertibility	51
3.4	Autoregressive and Mixed Processes.....	56
3.5	Problems and Solutions	68
	References.....	75
4	Spectra of Stationary Processes	77
4.1	Summary	77
4.2	Definition and Interpretation	77
4.3	Filtered Processes	84
4.4	Examples of ARMA Spectra	89
4.5	Problems and Solutions	95
	References.....	101

5	Long Memory and Fractional Integration	103
5.1	Summary	103
5.2	Persistence and Long Memory	103
5.3	Fractionally Integrated Noise	108
5.4	Generalizations	113
5.5	Problems and Solutions	118
	References	125
6	Processes with Autoregressive Conditional Heteroskedasticity (ARCH)	127
6.1	Summary	127
6.2	Time-Dependent Heteroskedasticity	127
6.3	ARCH Models	130
6.4	Generalizations	135
6.5	Problems and Solutions	142
	References	148
 Part II Stochastic Integrals		
7	Wiener Processes (WP)	151
7.1	Summary	151
7.2	From Random Walk to Wiener Process	151
7.3	Properties	157
7.4	Functions of Wiener Processes	161
7.5	Problems and Solutions	170
	References	177
8	Riemann Integrals	179
8.1	Summary	179
8.2	Definition and Fubini's Theorem	179
8.3	Riemann Integration of Wiener Processes	183
8.4	Convergence in Mean Square	186
8.5	Problems and Solutions	190
	References	197
9	Stieltjes Integrals	199
9.1	Summary	199
9.2	Definition and Partial Integration	199
9.3	Gaussian Distribution and Autocovariances	202
9.4	Standard Ornstein-Uhlenbeck Process	204
9.5	Problems and Solutions	207
	Reference	211
10	Ito Integrals	213
10.1	Summary	213
10.2	A Special Case	213
10.3	General Ito Integrals	218

10.4 (Quadratic) Variation	222
10.5 Problems and Solutions	229
References	237
11 Ito's Lemma	239
11.1 Summary	239
11.2 The Univariate Case	239
11.3 Bivariate Diffusions with One WP	245
11.4 Generalization for Independent WP	250
11.5 Problems and Solutions	254
Reference	258
 Part III Applications	
12 Stochastic Differential Equations (SDE)	261
12.1 Summary	261
12.2 Definition and Existence	261
12.3 Linear Stochastic Differential Equations	265
12.4 Numerical Solutions	272
12.5 Problems and Solutions	273
References	282
13 Interest Rate Models	285
13.1 Summary	285
13.2 Ornstein-Uhlenbeck Process (OUP)	285
13.3 Positive Linear Interest Rate Models	288
13.4 Nonlinear Models	292
13.5 Problems and Solutions	296
References	302
14 Asymptotics of Integrated Processes	303
14.1 Summary	303
14.2 Limiting Distributions of Integrated Processes	303
14.3 Weak Convergence of Functions	310
14.4 Multivariate Limit Theory	317
14.5 Problems and Solutions	321
References	329
15 Trends, Integration Tests and Nonsense Regressions	331
15.1 Summary	331
15.2 Trend Regressions	331
15.3 Integration Tests	336
15.4 Nonsense Regression	341
15.5 Problems and Solutions	344
References	352

16 Cointegration Analysis	353
16.1 Summary	353
16.2 Error-Correction and Cointegration	353
16.3 Cointegration Regressions	358
16.4 Cointegration Testing	365
16.5 Problems and Solutions	373
References	381
References	383
Index	389