

# CONTENTS

---

---

	PAGE
Preface . . . . .	v
Acknowledgments . . . . .	vii
SECTION	
0. Prerequisites . . . . .	1
CHAPTER I: SETS AND CLASSES	
1. Set inclusion . . . . .	9
2. Unions and intersections . . . . .	11
3. Limits, complements, and differences . . . . .	16
4. Rings and algebras . . . . .	19
5. Generated rings and $\sigma$ -rings . . . . .	22
6. Monotone classes . . . . .	26
CHAPTER II: MEASURES AND OUTER MEASURES	
7. Measure on rings . . . . .	30
8. Measure on intervals . . . . .	32
9. Properties of measures . . . . .	37
10. Outer measures . . . . .	41
11. Measurable sets . . . . .	44
CHAPTER III: EXTENSION OF MEASURES	
12. Properties of induced measures . . . . .	49
13. Extension, completion, and approximation . . . . .	54
14. Inner measures . . . . .	58
15. Lebesgue measure . . . . .	62
16. Non measurable sets . . . . .	67
CHAPTER IV: MEASURABLE FUNCTIONS	
17. Measure spaces . . . . .	73
18. Measurable functions . . . . .	76

SECTION	PAGE
19. Combinations of measurable functions . . . . .	80
20. Sequences of measurable functions . . . . .	84
21. Pointwise convergence . . . . .	86
22. Convergence in measure. . . . .	90
CHAPTER V: INTEGRATION	
23. Integrable simple functions . . . . .	95
24. Sequences of integrable simple functions . . . . .	98
25. Integrable functions . . . . .	102
26. Sequences of integrable functions. . . . .	107
27. Properties of integrals . . . . .	112
CHAPTER VI: GENERAL SET FUNCTIONS	
28. Signed measures . . . . .	117
29. Hahn and Jordan decompositions . . . . .	120
30. Absolute continuity . . . . .	124
31. The Radon–Nikodym theorem . . . . .	128
32. Derivatives of signed measures . . . . .	132
CHAPTER VII: PRODUCT SPACES	
33. Cartesian products . . . . .	137
34. Sections . . . . .	141
35. Product measures . . . . .	143
36. Fubini's theorem . . . . .	145
37. Finite dimensional product spaces . . . . .	150
38. Infinite dimensional product spaces . . . . .	154
CHAPTER VIII: TRANSFORMATIONS AND FUNCTIONS	
39. Measurable transformations . . . . .	161
40. Measure rings . . . . .	165
41. The isomorphism theorem . . . . .	171
42. Function spaces . . . . .	174
43. Set functions and point functions. . . . .	178
CHAPTER IX: PROBABILITY	
44. Heuristic introduction . . . . .	184
45. Independence . . . . .	191
46. Series of independent functions . . . . .	196

CONTENTS

xi

SECTION	PAGE
47. The law of large numbers . . . . .	201
48. Conditional probabilities and expectations . . . . .	206
49. Measures on product spaces . . . . .	211

CHAPTER X: LOCALLY COMPACT SPACES

50. Topological lemmas . . . . .	216
51. Borel sets and Baire sets . . . . .	219
52. Regular measures . . . . .	223
53. Generation of Borel measures . . . . .	231
54. Regular contents . . . . .	237
55. Classes of continuous functions . . . . .	240
56. Linear functionals . . . . .	243

CHAPTER XI: HAAR MEASURE

57. Full subgroups . . . . .	250
58. Existence . . . . .	251
59. Measurable groups . . . . .	257
60. Uniqueness . . . . .	262

CHAPTER XII: MEASURE AND TOPOLOGY IN GROUPS

61. Topology in terms of measure . . . . .	266
62. Weil topology . . . . .	270
63. Quotient groups . . . . .	277
64. The regularity of Haar measure . . . . .	282
References . . . . .	291
Bibliography . . . . .	293
List of frequently used symbols . . . . .	297
Index . . . . .	299