

Contents

1	Introduction and Summary	1
2	Equations of Classical Hydrodynamics	11
2.1	Derivation of the Equations of Motion	11
2.2	The Stress Tensor.....	20
2.3	Field Equations	22
2.4	Navier–Stokes Equations	23
2.5	Vorticity Dynamics	24
2.6	Thermodynamics	26
2.7	Similarity of Flows and Nondimensional Variables	28
2.8	Examples of Simple Exact Solutions	31
2.9	Comments and Bibliographical Notes.....	36
3	Mathematical Preliminaries	39
3.1	Theorems from Functional Analysis	39
3.2	Sobolev Spaces and Distributions	44
3.3	Some Embedding Theorems and Inequalities.....	48
3.4	Sobolev Spaces of Periodic Functions	54
3.5	Evolution Spaces and Their Useful Properties.....	61
3.6	Gronwall Type Inequalities	67
3.7	Clarke Subdifferential and Its Properties.....	70
3.8	Nemytskii Operator for Multifunctions	74
3.9	Clarke Subdifferential: Examples	77
3.10	Comments and Bibliographical Notes.....	81
4	Stationary Solutions of the Navier–Stokes Equations	83
4.1	Basic Stationary Problem	83
4.1.1	The Stokes Operator.....	84
4.1.2	The Nonlinear Problem.....	86
4.1.3	Other Topological Methods to Deal with the Nonlinearity.....	90
4.2	Comments and Bibliographical Notes.....	93

5	Stationary Solutions of the Navier–Stokes Equations with Friction	95
5.1	Problem Formulation.....	95
5.2	Friction Operator and Its Properties	96
5.3	Weak Formulation	98
5.4	Existence of Weak Solutions for the Case of Linear Growth Condition	103
5.5	Existence of Weak Solutions for the Case of Power Growth Condition	107
5.6	Comments and Bibliographical Notes.....	109
6	Stationary Flows in Narrow Films and the Reynolds Equation	111
6.1	Classical Formulation of the Problem	111
6.2	Weak Formulation and Main Estimates	114
6.3	Scaling and Uniform Estimates.....	121
6.4	Limit Variational Inequality, Strong Convergence, and the Limit Equation	125
6.5	Remarks on Function Spaces	128
6.6	Strong Convergence of Velocities and the Limit Equation	134
6.7	Reynolds Equation and the Limit Boundary Conditions	137
6.8	Uniqueness	141
6.9	Comments and Bibliographical Notes.....	142
7	Autonomous Two-Dimensional Navier–Stokes Equations	143
7.1	Navier–Stokes Equations with Periodic Boundary Conditions	143
7.2	Existence of the Global Attractor: Case of Periodic Boundary Conditions.....	151
7.3	Convergence to the Stationary Solution: The Simplest Case.....	157
7.4	Convergence to the Stationary Solution for Large Forces	159
7.5	Average Transfer of Energy.....	163
7.6	Comments and Bibliographical Notes.....	166
8	Invariant Measures and Statistical Solutions.....	169
8.1	Existence of Invariant Measures.....	169
8.2	Stationary Statistical Solutions	176
8.3	Comments and Bibliographical Notes.....	181
9	Global Attractors and a Lubrication Problem	183
9.1	Fractal Dimension	183
9.2	Abstract Theorem on Finite Dimensionality and an Algorithm....	185
9.3	An Application to a Shear Flow in Lubrication Theory	193
9.3.1	Formulation of the Problem	193
9.3.2	Energy Dissipation Rate Estimate	196
9.3.3	A Version of the Lieb–Thirring Inequality	200
9.3.4	Dimension Estimate of the Global Attractor	201
9.4	Comments and Bibliographical Notes.....	204

10 Exponential Attractors in Contact Problems	207
10.1 Exponential Attractors and Fractal Dimension	207
10.2 Planar Shear Flows with the Tresca Friction Condition	211
10.2.1 Problem Formulation	211
10.2.2 Existence and Uniqueness of a Global in Time Solution ..	218
10.2.3 Existence of Finite Dimensional Global Attractor	222
10.2.4 Existence of an Exponential Attractor	229
10.3 Planar Shear Flows with Generalized Tresca Type Friction Law	231
10.3.1 Classical Formulation of the Problem	231
10.3.2 Weak Formulation of the Problem	233
10.3.3 Existence and Properties of Solutions	236
10.3.4 Existence of Finite Dimensional Global Attractor	241
10.3.5 Existence of an Exponential Attractor	246
10.4 Comments and Bibliographical Notes.....	248
11 Non-autonomous Navier–Stokes Equations and Pullback Attractors	251
11.1 Determining Modes	251
11.2 Determining Nodes.....	256
11.3 Pullback Attractors for Asymptotically Compact Non-autonomous Dynamical Systems	260
11.4 Application to Two-Dimensional Navier–Stokes Equations in Unbounded Domains	268
11.5 Comments and Bibliographical Notes.....	274
12 Pullback Attractors and Statistical Solutions	277
12.1 Pullback Attractors and Two-Dimensional Navier–Stokes Equations	277
12.2 Construction of the Family of Probability Measures	281
12.3 Liouville and Energy Equations	285
12.4 Time-Dependent and Stationary Statistical Solutions	288
12.5 The Case of an Unbounded Domain	291
12.6 Comments and Bibliographical Notes.....	295
13 Pullback Attractors and Shear Flows	297
13.1 Preliminaries.....	297
13.2 Formulation of the Problem	298
13.3 Existence and Uniqueness of Global in Time Solutions.....	301
13.4 Existence of the Pullback Attractor	305
13.5 Fractal Dimension of the Pullback Attractor.....	310
13.6 Comments and Bibliographical Notes.....	316
14 Trajectory Attractors and Feedback Boundary Control in Contact Problems	317
14.1 Setting of the Problem	317
14.2 Weak Formulation of the Problem.....	319

14.3	Existence of Global in Time Solutions	322
14.4	Existence of Attractors	329
14.5	Comments and Bibliographical Notes.....	335
15	Evolutionary Systems and the Navier–Stokes Equations	337
15.1	Evolutionary Systems and Their Attractors	337
15.2	Three-Dimensional Navier–Stokes Problem with Multivalued Friction	340
15.3	Existence of Leray–Hopf Weak Solution	342
15.4	Existence and Invariance of Weak Global Attractor, and Weak Tracking Property.....	351
15.5	Comments and Bibliographical Notes.....	356
16	Attractors for Multivalued Processes in Contact Problems.....	359
16.1	Abstract Theory of Pullback \mathcal{D} -Attractors for Multivalued Processes.....	359
16.2	Application to a Contact Problem	366
16.3	Comments and Bibliographical Notes.....	376
References.....		377
Index.....		387